The Impact of Experts and Error in Observation on Informational Cascades.

Abstract:

Often models are used to study Bayesian agents where limited information is available. Models, in which these agents sequentially decide to accept or reject an option while observing previous agent's decisions, can produce *informational cascades*. An informational cascade is said to occur when an agent relinquishes their own private information in exchange for a pattern evident in the decisions of previous agents. The model in this paper has two types of agents each with differing signal accuracies: experts and non-experts. The type of agent is publicly known. The model incorporates noise in an identical manner as Le, Subramanian, and Berry (2014). With this model, we studied the probability of a correct cascade with respect to the inputted signal accuracies, noise, and expert concentrations. The probability of a correct cascade and noise showed a non-monotonic relationship characterized by "spikes" which appeared across the graph. The graphs of the probability of a correct cascade against expert concentration also showed that despite agents with better signal accuracies becoming more frequent, the likeliness of a correct cascade was also non-monotonic. Overall we showed that a lower noise and a higher expert concentration does not necessarily increase the probability of a correct cascade.

Introduction

Figure a scenario where agents line up to buy a product. Agents decide to buy or not by factoring in two components: their private knowledge and their observations of earlier agents. Bikhchandani, Hirshleifer, and Welch (1992)(BHW) uncovered the possibility that agents may find it optimal to follow previous agents even if their own signal may disagree. This phenomenon, where agents relinquish their own private information in exchange for the decision of their predecessors, is coined an *informational cascade*. Not only does an informational cascade cause a loss of information but it also creates the possibility of a wrong cascade.

After the BHW model in 1992, many variations stemmed out. Sasaki (2005) tested the effect of the order of the agents on the model. She organized them in two ways: seniority (highest to lowest signal accuracy) and anti-seniority (lowest to highest signal accuracy). She found that although seniority has more complete cascades, it is more likely to cause a wrong cascade. Pastine and Pastine (2006) altered the BHW model such that the signal accuracies were no longer symmetric. They showed that the slightest variation in its symmetry causes signal accuracy to have a non-monotonic effect on the probability of an inefficient cascade. The model discussed in this paper is completely symmetrical.

Wu (2015) extended the BHW model to incorporate two types of agents: experts and laymen. Experts had higher-signal accuracy than laymen, but their type remained anonymous to the population. Her model shows that a mix between agents is strictly better than a homogenous sample. The model discussed in this paper has two agent types as well: expert and non-experts. Similarly, experts have higher signal accuracy. This model differs from Wu (2015) as the identities of the experts are known and have a larger influence on an agent than a non-expert.

Le, Subramanian, and Berry investigated the impact of errors in the observations of previous agents on total payoff. They concluded that a lower error level does not always mean a higher payoff. The model discussed in this paper also incorporates noise in a nearly identical fashion as Le, Subramanian, and Berry.

This model studies the $Pr(Correct\ Cascade)$ by varying inputted parameter in a Monte Carlo simulation. These parameters include P(non-expert signal accuracy), P_{exp} (expert signal accuracy), Q (fraction of experts), ε (error in observation). Graphs of $Pr(Correct\ Cascade)$ against Q exhibits a wide range of graphs some of which are non-monotonic, monotonic, U-Shaped, or almost flat (see Figure A). The graphs of $Pr(Correct\ Cascade)$ and ε was also non-monotonic having occasional "spikes" (see Figure B). By comparing the graphs to fluctuation in the model's parameters, some of these spikes can be explained. The model also showed that $\varepsilon = 0$ does not always produce the max $Pr(Correct\ Cascade)$.
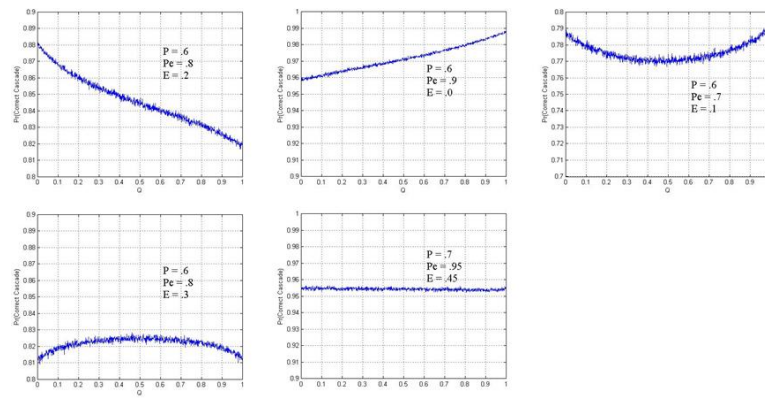

Figure A: The Figure Above depict 5 different shapes of Pr(Correct Cascade) against Q with their P, P_{exp}, and ε displayed on the grid. Interestingly as more agents with better signal accuracies appear more often the $Pr(Correct\ Cascade)$ does not always increase.

Figure B: The figure above shows $Pr(Correct\ Cascade)$ against ε, the error in observation. For this specific run $P = 0.6, P_{exp} = 0.8\ and\ Q = 0.2$. The graph shows that for some scenarios, $\varepsilon = 0$ is not the global
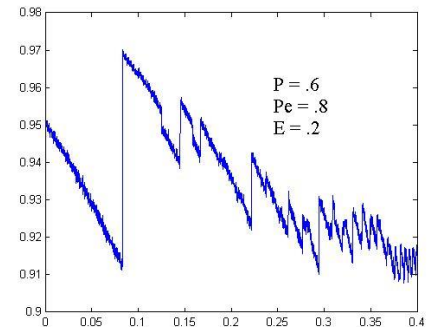
This paper is as organized as followed. Section II discusses the model set up and necessary calculations. We study certain patterns is Section III and conclude in Section IV.

Section II: Model Setup

The model assumes the state of the object, V, is either G or B both with probability ½. There are an infinite number of agents, listed $i = 1,2,3$ ..., in an exogenous order. An agent is an expert with probability $Q \in (0,1]$. Non-experts have a signal accuracy of $P \in (0.5,1)$, and experts have a signal accuracy of $P_{exp} \in (0.5,1)$ given that $P_{exp} \geq P$. The variable, $\varepsilon \in [0,0.5)$, is the probability that an agent incorrectly observes the previous agent's decision.

Agents' observations of their predecessors are documented through a Markov Chain. A non-expert's approval increases the states of the Markov Chain by $njump$ with probability a, and an expert's decision moves up $ejump$ with a probability b. The model is symmetric for when the agent disagrees (See Figure C).

The variable, a, is the probability that the agent$_{i+1}$ correctly observed agent$_i$ and agent$_i$ accepted plus the probability the agent$_{i+1}$ incorrectly observed agent$_i$ and agent$_i$ rejected.

$$a = P(1 - \varepsilon) + (1 - P)\varepsilon$$

With the same logic:

$$b = P_{exp}(1 - \varepsilon) + (1 - P_{exp})\varepsilon$$

The variables, $ejump$ and $njump$, must be set up such that when the Markov Chain is at 0, the agent should be completely indifferent. Thus:

$$\Pr\left(V = G | Y_{expert_i}, Y_{expert_{ii}} \dots Y_{expert_{njump}}, N_{normal_i}, N_{normal_{ii}} \dots N_{normal_{ejump}}\right) = \frac{1}{2}$$

By Bayes Rule:

$$\Pr\left(Y_{expert} | S = G\right)^{njump} * \Pr(N_{normal}|G)^{ejump}$$
$$= \Pr\left(Y_{expert} | S = B\right)^{njump} * \Pr(N_{normal}|B)^{ejump}$$

By substitution:

$$b^{njump} * (1 - a)^{ejump} = a^{ejump} * (1 - b)^{njump}$$

Resulting in:

$$\log_{\frac{a}{1-a}} \frac{b}{1-b} = \frac{ejump}{njump}$$

By inputting values of a and b computed by the inputted parameters, the model finds integer values for $ejump$ and $njump$ to fit the formula above.

The formula above is also used to find the probability the chain will move up by 1. This is probability is denoted by ϑ and is later used.

$$\frac{b}{1-b}^{\frac{1}{ejump}} = \frac{a}{1-a}^{\frac{1}{njump}} = \frac{\vartheta}{1-\vartheta}$$

The Markov Chain has 4 trapping states each representing the start of a partial or full cascade: $-K_{exp}, -K, K, K_{exp}$ (See Figure C). States $-K$ and K trap all non-experts creating a cascade of non-experts. However, this cascade can be broken by the influence of an expert. All agents, however, get stuck in the $-K_{exp}$ and K_{exp} trapping states; resultantly, the model ends when either one is reached (See Figure C).
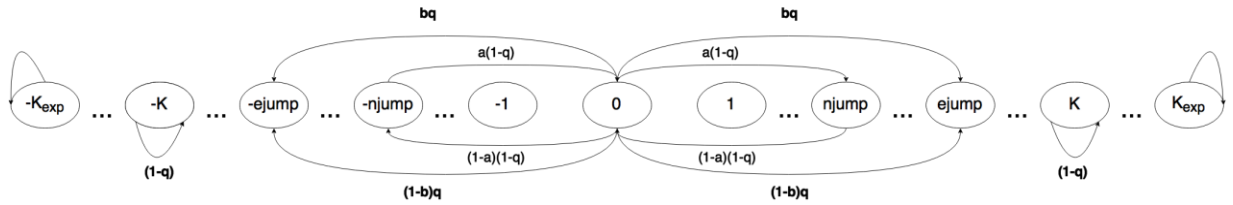

Figure C: Above is a graphical representation of the Markov Chain. Both K and K_{exp} are trapping states. However, only experts can move between K and K_{exp} (or between $-K_{exp}$ and $-K$). The probability of being an expert is Q. Experts move by $ejump$ and non-experts move by $njump$. The probability of moving up as an expert is b and the probability of moving up as a non-expert is a.

An agent decides to buy based on his private information and his observations of previous agents.

Thus, if (where F represents the current state of the Markov Chain and i_{signal} represents the signal of the agent)

$$P(V = G | S = F, i_{signal}) > \frac{1}{2} \; then \; the \; agent \; buys$$

$$P(V = G | S = F, i_{signal}) < \frac{1}{2} \; then \; the \; agent \; does \; not \; buy$$

$$P(V = G | S = F, i_{signal}) = \frac{1}{2} \; then \; the \; agent \; follows \; crowd$$

An informational cascade occurs when an agent forgoes their own signal in exchange for their predecessor's choices. We also know that informational cascades occur when the current state is K. Thus:

$$P(V = G, S = K, Signal = B) = \frac{1}{2}$$

By Bayes theorem:

$$\frac{P(V = G) * P(Signal = B, S = K | V = G)}{P(V = G) * P(Signal = B, S = K | V = G) + P(Signal = B, S = K | V = B) * P(V = B)}$$

Therefore:

$$P(V = G) * P(Signal = B, S = K | V = G) = P(Signal = B, S = K | V = B) * P(V = B)$$

Through substitution

$$(1 - p) * \vartheta^k = p * (1 - \vartheta)^k$$

$$K = \log_{\frac{\vartheta}{1-\vartheta}} \frac{p}{(1 - p)}$$

We can substitute P_{exp} to get K_{exp}.

$$K_{exp} = \log_{\frac{\vartheta}{1-\vartheta}} \frac{P_e}{(1 - P_e)}$$

Initially we used First-Step Analysis to determine the exact $Pr(Correct\ Cascade)$: let $\pi(j)$ be the probability that an agent will go from state j to K_{exp}. $\pi(j)$ can be written in terms of possible states an agent can travel to from j. For example:

$$\pi(0) = a(1-q) * \pi(njump) + (1-a)(1-q) * \pi(-njump) + b * q * \pi(ejump) + (1-b) * q * \pi(-ejump)$$

When j nears K or K_{exp}, the number of states an agent can travel to get limited. By putting $\pi(j)$ for all $j \in (-K_{exp}, K_{exp})$, in terms of other states, we can use MatLab to solve a larger system of equations and output $\pi(0)$ or in other words, $Pr(Correct\ Cascade)$. However due to the limitations of MatLab, as K_{exp} increases, MatLab is incapable to solve the systems of equations. As an alternative, we use Monte Carlo to run a simulation. With many runs, the Monte Carlo simulation serves as a precise estimation.

Section III: Results

In order to analyze the model, we graphed $Pr(correct\ cascade)$ against ε across various Q, P, and P_{exp}. We also graphed $Pr(Correct\ Cascade)$ against Q. In these graphs, three major phenomena arose.

Pr(Correct Cascade) against Q Graph Shapes

Without analysis, one would think with additional experts, $Pr(Correct\ Cascade)$ should continually rise. However, simulations runs show various graph shapes (See Figure A). The general shape of the graph was compared to certain components of the simulation: $\left\lceil \frac{K}{njump} \right\rceil$, $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$, and CDBTS (Corrected Distance Between Trapping States). No patterns were found. We can then conclude that the varied shapes are due to a combination of factors and cannot be predicted.

Spikes in Pr(Correct Cascade) against ε

As the noise in the simulation increases, there are spikes in the graphs (See Figure B). In order to explain these spikes we overlaid key components of the graphs as they change with E in order to see any correlation. We compared the graphs with $\left\lceil \frac{K}{njump} \right\rceil$, $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$ and the CDBTS (See Figure

D). By doing so some of the spikes could be explained, but the cause of others are unknown. In likelihood it is a combination of factors which causes those spikes. As $|(\varepsilon - .5)|$ gets closer to zero, there are more spikes that cannot be explained (See Figure D). This is likely caused by the presence of two agent types which makes the simulation more complex. It was also noticed that when Q is small, most spikes are explained by $\left\lceil \frac{K}{njump} \right\rceil$, but when Q larger most of the spikes are explained by $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$. Since $\left\lceil \frac{K}{njump} \right\rceil$ is the amount of normal agents required to get past the first trapping state, it makes sense that when majority of agents are non-experts (when Q is small) $\left\lceil \frac{K}{njump} \right\rceil$ would have a larger effect (See Figure E). Similarly, when majority of agents are experts (Q is larger), $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$ would have a larger affect (See Figure F).
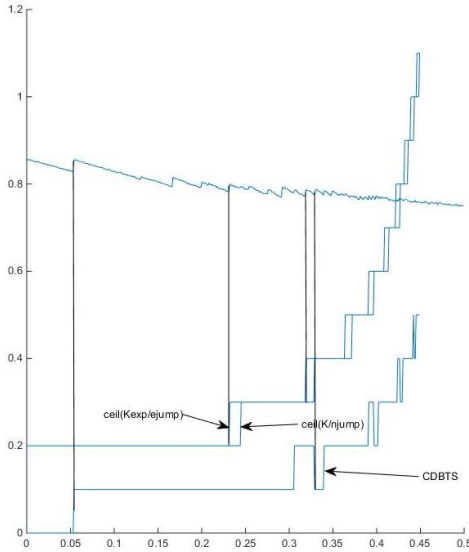

Figure D: The figure above shows $\left\lceil \frac{K}{njump} \right\rceil$, $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$, **and the CDBTS as they fluctuate with epsilon. It also shows** $Pr(Correct\ Cascade)$ **with the input parameters:** $P = 0.65, P_{exp} = 0.75,$ **and** $Q = 0.5.$ **As** Q **is 0.5, the duality of the agents has the largest effect. This causes spikes to be caused by a mixture of factors and cannot be attributed to just** $\left\lceil \frac{K}{njump} \right\rceil$, $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$ **or CDBTS. Note** $\left\lceil \frac{K}{njump} \right\rceil$, $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$ **or CDBTS are all divided by 10 to better fit on the graph.**
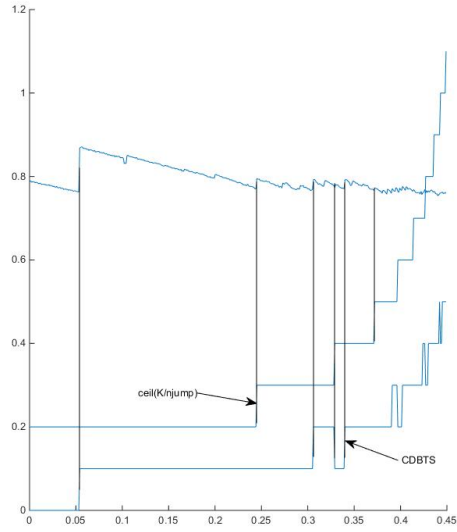
Figure E: The figure above shows $\left\lceil \frac{K}{njump} \right\rceil$ **and CDBTS as they fluctuate with epsilon. It also shows** $Pr(Correct\ Cascade)$ **with the input parameters:** $P = 0.65$, $P_{exp} = 0.75$, **and** $Q = 0.05$. **As** Q **is 0.05, majority of agents are non-experts. This causes most of the spikes to be explained by to be caused by** $\left\lceil \frac{K}{njump} \right\rceil$ **which affect only non-experts. Note** $\left\lceil \frac{K}{njump} \right\rceil$ **and CDBTS are both divided by 10 to better fit on the graph.**
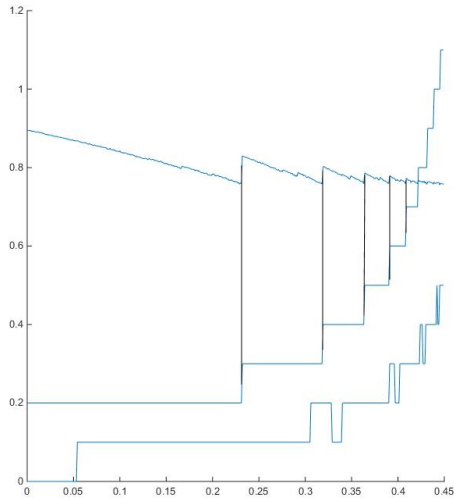

Figure F: The figure above shows $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$ **and CDBTS as they fluctuate with epsilon. It also shows** $Pr(Correct\ Cascade)$ **with the input parameters:** $P = 0.65$, $P_{exp} = 0.75$, **and** $Q = 0.95$. **As** Q **is 0.95, majority of agents are experts. This causes most of the spikes to be explained by to be caused by** $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$ **which affect only experts. Note** $\left\lceil \frac{K_{exp}}{ejump} \right\rceil$ **and CDBTS are both divided by 10 to better fit on the graph.**

Pr(Correct Cascades) against ε when $\varepsilon = 0$

For some graphs, $\varepsilon = 0$ is not the highest $Pr(Correct\ Cascade)$. In other words, in some cases having perfect observation is less effective than having some error in observation. Again we

overlaid $\left|\frac{K}{njump}\right|$, $\left|\frac{K_{exp}}{ejump}\right|$, and CDBTS to determine the factors which caused the first spike. In all cases when $Pr(Correct\ Cascade)$ was not the highest point, the first spike was caused by an increase in CDBTS (See Figure G). Essentially, if the distance between trapping states increases before the noise becomes too large, $Pr(Correct\ Cascade)$ will be higher for some $\varepsilon > 0$. This shows that an additional state between the trapping states outweighs the effect the noise had on the simulation.
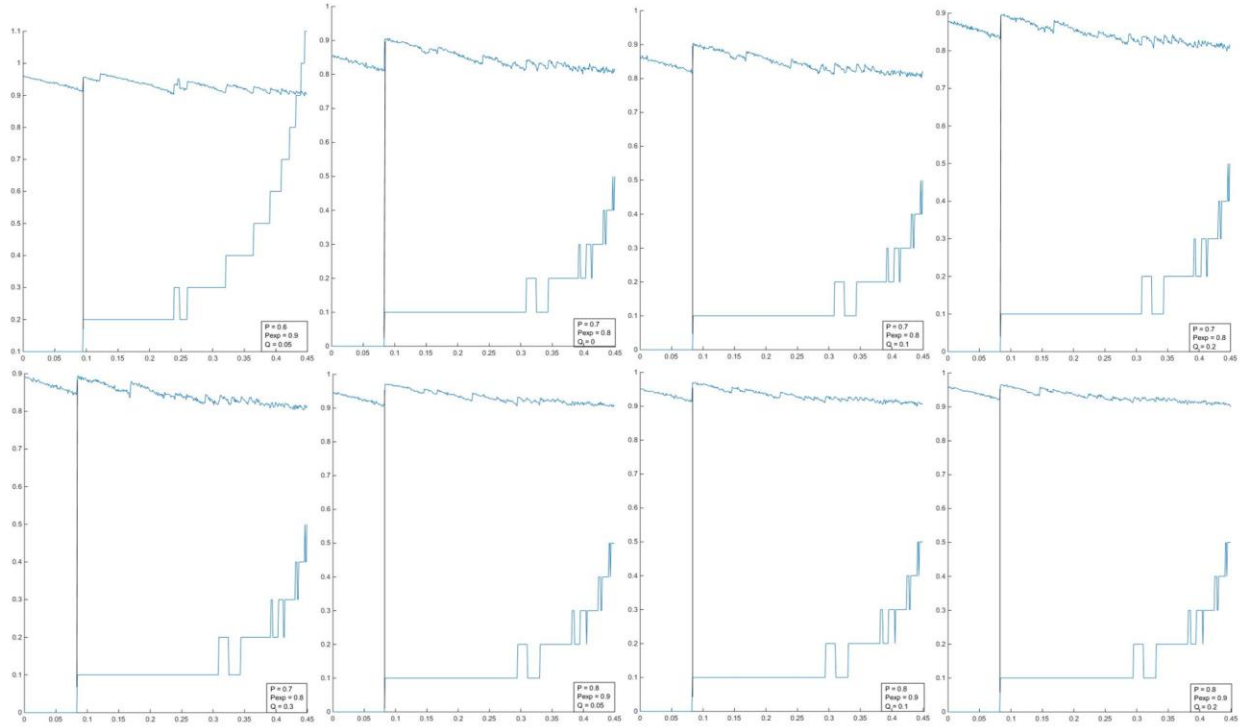

Figure G: The figure above shows CDBTS as it fluctuates with ε. With all of the presented cases for some $\varepsilon > 0$, $Pr(Correct\ Cascade)$ is higher than it was at $\varepsilon = 0$. The graph of CDBTS shows that the first spike always corresponds with the first spike in $Pr(Correct\ Cascade)$. We conclude that if the CDBTS increments with a relatively low ε, then the benefit of an extra state between the two trapping states may outweigh the negatives of the additional noise causing a global maximum at an $\varepsilon > 0$.

Conclusion

This paper studied the effect of expert concentration and error in observation in a simple informational cascade model. By adding noise in a similar fashion as Le, Subramanian, and Berry, and creating two types of agents with independent signal accuracies, we determined the probability of a correct cascade using First-Step Analysis. Due to limitation with size of the Markov Chain, we used Monte Carlo simulations to determine a precise estimation

of $Pr(Correct\ Cascade)$. Overall the main results shows that an increase in expert concentration or a decrease in noise does not necessarily increase the $Pr(Correct\ Cascade)$ and in certain cases $\varepsilon = 0$ is not the highest $Pr(Correct\ Cascade)$. Thus there is benefit of having a mixture of experts and non-experts and noise in some cases. Possible extensions include exploring when agent types are not publicly known and when experts are harder to be swayed by the public than a non-expert.

Appendix

Correct Distance Between Trapping States (CDBTS)

$$\left| \frac{Kexp - \lceil K/njump \rceil njump}{ejump} \right|$$

Works Cited

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